Transition–Based Budgeting
To Improve Salary Projections:
The Case Of Education

JAMES N. FOX

The current state-of-the-art for projecting school budgets is extremely primitive. Hentschke offers some general principles to consider when constructing budget projections, but offers no explicit guidance beyond that general advice.¹ Candoli and his colleagues state that “salary projections for general budgeting purposes are little better than guesses. . . .”² This state of affairs was recently confirmed by a case study of school budget projections which demonstrated that standard forecasting techniques miss the mark by 5 percent.³ In larger school districts, this results in errors amounting to millions of dollars.

Teacher pay is the major item in school budgets. About 40 percent of all school spending goes to support teachers.⁴ Spending on this line item is particularly difficult to project. Teachers tend to transfer to another district, move into school administration, or quit teaching altogether. The National Governors’ Association estimates that in a number of districts up to 40 percent of all entering teachers quit the profession during their first two years of teaching.⁵ Traditional school budget projection techniques that do not carefully consider such factors are likely to miss the mark.

THE RATIONALE

Taylor has demonstrated that Markov analysis may be used to project teacher salaries.\(^6\) Presented below is an alternative methodology for increasing the accuracy of school budget projections. This approach, called transition–based budgeting, uses techniques from Markov analysis but is far simpler and more straightforward than the traditional Markov approach.\(^7\) Also, as demonstrated below, transition–based budgeting may yield far more accurate budget projections than those generated by Markov analysis.

The use of transition–based budgeting rather than Markov analysis is justified by the presumption that school managers operate in a far more predictable environment than do managers of competitive firms in the business sector. Markov analysis may be relevant for determining the equilibrium market share of firms competing in the private sector,\(^8\) but it may be overly complex for projecting teachers' salaries. School budget officers face a fairly predictable demand for teachers, and tenure provisions and other information provide them with relatively accurate data regarding the supply of personnel available to them in the near future. In short, school officials are far less subject to the vagaries of supply and demand than are the managers of the typical competitive firm. As a result, it seems to make sense to use a straightforward approach such as transition–based budgeting, rather than an approach based on a state of equilibrium, such as Markov analysis.

This paper describes transition–based budgeting, discusses some problems with Taylor's approach which uses Markov analysis to project teachers' salaries, and then contrasts transition–based budgeting with Markov analysis.

THE APPROACH

The first step in transition–based budgeting is to develop the Transition Probability Matrix. The rows of this matrix reflect the current status of teachers (new, continuing, on sabbatical, on leave or ill). The columns of the matrix reflect the status each type of teacher would enter during the next period under consideration.

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6. Taylor and Reid, op. cit.
7. Specifically, transition–based budgeting is an application of the transition matrix that undergirds Markov analysis. However, as opposed to projections from traditional Markov analysis, transition–based projections are founded on the probabilities from the Transition Probability Matrix itself. The probabilities are not taken from the Steady State Probability Vector, as is the case in Markov analysis.
The numbers in cells represent the probabilities of teachers moving from one state to another. For the purposes of constructing a transition-based budget projection, transitions from the system (resign, retire, die) need not be calculated. However, to increase accuracy, such calculations might be beneficial. Then the sum of probabilities in each row must equal unity because all teachers must move into one of the possible alternative states.

The probabilities should reflect actual transitions as closely as possible. The degree to which this can be accomplished will depend in large part on the timing of the budget projection. For example, if the budget for the 1986–87 school year were projected at the beginning of the 1985–86 school year, the probability for a transition from, continuing to (remain as a) continuing teacher might be based on a three year running average of this transition. That average might then be adjusted upward if more teachers than usual were scheduled to reach the mandatory or usual retirement age, or downward if fewer teachers than usual fall into this category at the end of the school year. Then, if the budget projection were revised near the end of the 1985–86 school year, such probabilities could be modified to reflect the numbers of teachers actually planning to retire. Table 1 displays a Transition Probability Matrix using data from a case study of budget projections.9

In Step 2 the Initial Status Vector is determined. This identifies the number of teachers in each state at the time the budget projections are developed. School budget officers accomplish this by simply tallying the number of employees in each state. This vector considers only those states that contribute to current operating costs: continuing, new, on leave, on sabbatical, and ill. This reflects the view that teachers who retire, resign or die contribute to current operating costs only by opening up a position to be filled (usually by a new teacher).

In Step 3, the number of teachers in each state for the upcoming budget year is estimated. This involves, first, multiplying the number of teachers in each initial state by the appropriate transformation probability, yielding the estimated number that will be in each state in the upcoming year. The total of these estimates is the number of positions that will be filled in the upcoming year. The difference between this total and the number of positions

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9. The case study did not directly provide the number of teachers in each initial state. This was determined by multiplying the total number of teachers in the case study system—258—by the initial condition probabilities provided in Table 2 of the case study. See Taylor and Reid, “Forecasting Salaries.”
<table>
<thead>
<tr>
<th></th>
<th>In the System</th>
<th></th>
<th>Leave the System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One Year Experience</td>
<td>Continuing</td>
<td>On Leave</td>
</tr>
<tr>
<td>(FROM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New</td>
<td>0.773</td>
<td>—</td>
<td>0.000</td>
</tr>
<tr>
<td>Continuing</td>
<td>—</td>
<td>0.896</td>
<td>0.022</td>
</tr>
<tr>
<td>On leave</td>
<td>—</td>
<td>0.178</td>
<td>0.289</td>
</tr>
<tr>
<td>Sabbatical</td>
<td>—</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Ill</td>
<td>—</td>
<td>0.906</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Source: Adapted from Raymond G. Taylor and William Michael Reid, "Forecasting the Salaries of Professional Personnel: An Application of Markov Analysis to School Finance," *Journal of Education Finance*, vol. 12, no. 2 (Fall 1987), Table 1, p. 425.
needed yields the number of positions to be filled. (Typically, these are filled by new teachers.)

The final step in the process is multiplying the number of teachers expected in each state by the anticipated cost of each teacher in that state. The sum of these products yields the budget projection.

**Using the Tools: A Case Study**

A case study of school budgeting demonstrates the process. Case study data are shown in both the Transitional Probability Matrix (Table 1) and the Initial Status Vector (Table 2). For reasons noted above, Table 2 contains no entries for teachers who have retired, resigned, or died. These categories are considered to be intermediary; as such, they do not contribute directly to the current operating costs of the district.

Table 3 shows the number of personnel estimated to be in various states at the beginning of the 1983–86 school year. These figures are obtained by multiplying the number of teachers in the initial states by the appropriate transition probability. Seven transitions relevant to calculating the operating budget are examined: continuing teachers who continue; new teachers who become teachers with one year of experience; teachers who return from sabbatical or other leave; continuing teachers who return from being ill at the end of the 1983–85 school year; new teachers who were ill at the end of the prior year and who now return with one year of experience; and ill teachers who remain ill but continue as salaried employees of the district.

Table 2 projects that 201.4 positions will be filled by returning teachers. How many positions remain to be filled? The system in the case study supported a total of 226 “active” positions in the 1983–85 school year (the sum of teachers initially in the three active states: continuing, new, and ill—see Table 1). Because the case study mentions nothing about expansion or contraction of the total number of positions between the 1983–85 and 1985–86 school years, 226 will be used to represent the number of positions to be filled during 1985–86. If 201.4 of these positions will be filled by returning teachers, 24.6 positions remain to be filled. As in the case study, the assumption is that these positions will be filled by new teachers.

<table>
<thead>
<tr>
<th>TABLE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL STATUS</td>
</tr>
<tr>
<td>STATE</td>
</tr>
<tr>
<td>NUMBER OF TEACHERS</td>
</tr>
</tbody>
</table>

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121
TABLE 3
PROJECTED PERSONNEL

<table>
<thead>
<tr>
<th>Transition</th>
<th>Number in Initial State</th>
<th>Transitional Probability</th>
<th>Number in Transformed State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuing to Continuing</td>
<td>196 x</td>
<td>.896</td>
<td>175.62</td>
</tr>
<tr>
<td>New to One Year Experience</td>
<td>24 x</td>
<td>.773</td>
<td>18.55</td>
</tr>
<tr>
<td>Sabbatical to Continuing</td>
<td>1 x</td>
<td>1.000</td>
<td>1.00</td>
</tr>
<tr>
<td>On Leave to Continuing</td>
<td>6 x</td>
<td>.178</td>
<td>1.07</td>
</tr>
<tr>
<td>III (Continuing) to Continuing*</td>
<td>2 x</td>
<td>.806</td>
<td>1.61</td>
</tr>
<tr>
<td>III (New) to One Year Experience*</td>
<td>4 x</td>
<td>.806</td>
<td>3.22</td>
</tr>
<tr>
<td>III to II</td>
<td>6 x</td>
<td>.055</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td></td>
<td><strong>201.40</strong></td>
</tr>
</tbody>
</table>

*The calculations regarding teachers in the ill category assume that one-third of the ill teachers in 1984–85 (that is, two) came from the ranks of continuing teachers and two-thirds of them (that is, 4) were new teachers during the 1984–85 school year. This is consistent with the assumptions that underlie the calculations in the original case study. See Taylor and Reid, “Forecasting Salaries,” p. 427.

Finally, the projected budget is calculated by multiplying the anticipated number of teachers in each state by the cost of each teacher in that state. To the resulting base budget are added such ancillary costs as the costs of supporting teachers on sabbatical and the costs of paying substitutes when other teachers are ill. Such an approach focuses only on members of the teaching workforce who are still in the system, reflecting the view that only active members of the workforce (plus supplementary costs for illness and sabbaticals) contribute to current operating costs, and that teachers who retire, resign, or die contribute to current operating costs only by freeing up a position to be filled by a new teacher.

Budget calculations are shown in Table 4. The largest contribution to the budget (Line 1) comes from continuing teachers who stay in the system. The case study indicates that, on the average, members of this group receive the average teacher’s salary. Therefore the cost factor for each member of this group is 1.6.10

Line 2 refers to new teachers who stay in the system to become teachers with one year of experience. In actual practice the first year’s salary increment is taken from the district’s salary schedule (adjusted to incorporate estimates of any salary adjustments to account for extra training). This information was not included in the case study, so a figure of 4.49 percent was drawn from Monk and Jacobson’s study of teachers’ salaries.11 (The district in the

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case study employed more than 200 teachers and thus is best represented by salaries in Monk and Jacobson's "large districts," which gave their beginning teachers an average pay increment of 4.49 percent.) Applying the 4.49 percent increase to the base factor of 1.1 yields a new adjustment factor of 1.15, which is used to estimate the salary of teachers with one year of experience.

Lines 3 and 4 reflect the cost of teachers returning from sabbatical or leave. These teachers are assumed to earn average salaries. The next three lines portray the cost projections for teachers who were assigned the status "ill" during the 1984–85 school year. Case study data suggest that two-thirds of these ill teachers were new and one-third were continuing. Thus, two-thirds of the ill teachers who continue would move from "new" status to become teachers with one year of experience; as a result, they are assigned a salary of 1.15 times the base of $12,400. Similarly, one-third of the ill teachers who continue would maintain their status as continuing teachers. These teachers are assumed to earn 1.6 times $12,400. Teachers who remain ill are assumed to be in the same two-thirds/one-third split. Thus, the salary cost calculation for each of them is:

12. Taylor and Reid, op. cit.
\[(.67 \times 1.2 \times \$12,400) + (.33 \times 1.6 \times \$12,400) = \$16,517\]  \hspace{1cm} (1)

The last two lines represent the ancillary costs, including the costs of sabbaticals and substitutes. The number of teachers expected to go on sabbatical was calculated by applying the appropriate probability from the Transition Probability Matrix against the number of continuing teachers in the initial state \((196 \times .007)\). Also, because only continuing teachers go on sabbatical, the cost of supporting each teacher on sabbatical was calculated as one-half the average teacher's salary.\(^{13}\)

A final step followed an assumption of the authors of the case study: that the district would be required to purchase an average of thirty days of substitute teacher time for each teacher who was assigned to the ill category. The Transition Probability Matrix enables an estimate that 3.42 continuing teachers \((201.4 \times .017)\) and .81 new teachers \((24.6 \times .033)\) will be assigned to the "ill" category.

In actual practice, it is not necessary to assign teachers to a separate category for illness; the district simply estimates the cost of substitute teachers. This could be accomplished by calculating a running average of number of substitute days actually required over, for example, the past three years. This figure might be adjusted if a district finds that new teachers or teachers nearing retirement tend to take off more days than the average teacher, or if the district in the upcoming year has a disproportionate number of teachers in either category.

**AN ALTERNATIVE APPROACH**

Analyses have recently proposed using Markov Analysis to project school budgets. A case study indicates that Markov procedures produce budget projections that are almost three times more accurate than those produced by traditional techniques.\(^{14}\) However, the paper advancing this technique contains two major flaws.

First, the budget projection in the case study inexplicably mixes "cost expansion" factors with factors that contribute directly to a given year's budget projection.\(^{15}\) Specifically, the "costs" for teachers who resigned, retired, were on leave, or had died were calculated as the difference between the salary of the teacher leaving the system and the salary of a new replacement teacher.\(^{16}\)

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\(^{13}\) *Ibid.*


\(^{15}\) *Ibid.* Table 4.

In other words, these elements were considered to be expansions (actually contractions) **relative to the 1984–85 budget**. On the other hand, the cost of the other factors—new teachers, continuing teachers, teachers on sabbatical, and ill teachers—were calculated according to the **direct contribution** these items would make to the **1985–86 budget**. No explanation is advanced for this seeming switching of horses, back and forth, as one crosses the proverbial stream.

Logical consistency would seem to dictate that budget projections must be based on one of two approaches: (1) Start with a base year figure and project additions to or subtractions from that base; or (2) project the direct contributions to the budget for the year for which the projection is being constructed. The case study appears to indicate a switching from one approach to the other rather than a choosing of one or the other. If relative projections were being constructed, the base year figure should appear; for example, continuing teachers (who are assumed to be paid at the same rate as in the base year) would contribute nothing to the expansion or contraction of these base year costs, rather than contributing the $4,089,024 listed in Table 4 of the case study. If the direct projection approach were being employed, teachers who have retired, resigned, are on leave, or have died contribute nothing to current operating costs because they are no longer in the system. The authors of the analysis offer no justification for the seemingly contradictory combination of these two approaches.

The second problem (related to the first) is the failure to distinguish time periods clearly. In Markovian terms, the paper fails to clearly identify $T_0$, the time when members of each class are in their initial states, and $T_1$, the time when members of each class are in their fully transformed states. The authors state that the proportions of teachers who were “active” at the end of the first year (new teachers, continuing teachers, and ill teachers) were treated as the beginning set of states, and that “teachers were excluded who resigned, retired, were on sabbatical or leave, or who were deceased.” However, Table 2 of the case study, the Initial Condition Probability Vector, seems to contradict that proposition. The nonactive states (retired, resigned, on sabbatical or leave, deceased) are all assigned proportions greater than zero. This seems directly inconsistent.

The confusion may center around what constitutes $T_0$ and $T_1$. If $T_0$ is defined as the beginning of the school year, then no
retired or resigned teachers are in the system; they have all been replaced (typically by a new teacher). If, on the other hand, the end of the school year is viewed as $T_0$, the argument could be made that while a number of teachers have announced their resignations or retirement, there are no longer any "new" teachers in the system. The former "new" teachers have been transformed to either "one year experience" or some other status, and the new "new" teachers who will replace retiring or resigning teachers have yet to be hired. It seems apparent that to count both retiring teachers and resigning teachers as well as the teachers who will replace them is unjustifiable double counting.

The Bottom Line. If the Markov analysis is adjusted to take account of the concerns noted above, then a different budget projection is reached. $T_0$ is the beginning of the 1984–85 school year, and $T_1$ the beginning of the 1985–86 school year. The direct (as opposed to the expansive) method is used to calculate budget projections. Under this model, teachers who resign, retire, go on leave, or die make no direct impact on the budget. Rather, these are intermediate transitional states that make their impact on the budget only when they reach their fully transformed state, typically that of new teacher.

If "resigned," "retired," "on leave," and "deceased" are eliminated from Table 4 of the case study (also eliminating the problem of double counting), then the following budget projection results:

<table>
<thead>
<tr>
<th>Category</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>$267,344</td>
</tr>
<tr>
<td>Continuing</td>
<td>4,089,024</td>
</tr>
<tr>
<td>Sabbatical</td>
<td>32,984</td>
</tr>
<tr>
<td>Ill (adjusted)</td>
<td>70,844</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$4,460,196</strong></td>
</tr>
</tbody>
</table>

Under these conditions, the Markov projection is further off target than the projection of $4.425$ million that was generated by traditional methods. (Actual expenditures for teacher salaries for the 1985–86 school year were $4.228$ million.)

A Comparative Analysis

An appealing aspect of papers on topics such as budget projections is that the precision of proposed techniques can be immediately tested. A reasonable criterion for evaluation is: How accurately does a given technique forecast a particular budget? Table 5 contrasts the accuracy of projections using alternative forecasting techniques. For each of four approaches, Table 5 lists
the absolute deviation from the target (an actual 1985–86 budget of $4.228 million) as well as the percentage deviation from that target amount.

Table 5 shows that the Markov approach advanced in the case study is about three times more accurate than the traditional deterministic approach actually employed in the district where the case study was conducted. The corrected Markov approach is slightly less accurate than the deterministic approach.

Transition–based budgeting is far more accurate than either the traditional approach or the corrected Markov approach. The method actually employed in the case study district generated a budgeting error of $197,000. The corrected Markov approach led to an error of $232,195, and the transition–based budgeting missed the mark by only $667—on a budget of $4.228 million. In short, transition–based budgeting is more than 100 times as accurate as the originally proposed Markov technique and about 300 times more accurate than the deterministic approach or the corrected Markov approach.

Transition–based budgeting has a number of advantages over the Markov approach:

1. Markov analysis is applicable only when the transition probabilities are reasonably constant over time.\(^{18}\) This is not a condition for transition–based budgeting, whose forecasting procedures allow the transition probabilities to be adjusted as circumstances vary.

2. Markov analysis is best performed with the help of a computer; transition–based forecasts can be constructed on the back of an envelope.

3. The underlying assumptions and required procedures of transition–based forecasting are more readily understood than those of Markov analysis.

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4. Most importantly, transition–based projections seem to be far more accurate than projections based on Markov techniques.

SUMMARY

Teacher salaries constitute the largest item in school budgets, but these costs are particularly difficult to forecast because teachers tend to move, retire or resign. This paper presents a technique for forecasting teacher salaries. A case study demonstrates that this simple, straightforward technique, called transition–based budgeting, can increase the accuracy of budget projections by as much as 300 percent.